# NON-SMOOTH ANALYSIS IN A NATIONAL ELECTRICITY MARKET MODEL: A COMPLEX APPROACH

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## ABSTRACT.

In this paper, we show the preliminary results in a proposed a model for the supply and demand of electricity in a domestic market based on system dynamics. Additionally, the model indicates piecewise smooth differential equations arising from the diagram of flows and levels, using dynamical systems theory for the study of the stability of the equilibrium points that have such a system. A bifurcation analysis approach is proposed to define and

understand the complex behavior. Until now, no work has been reported related to this topic using bifurcations criteria. The growing interest in personal ways of self-generation using renewable sources can lead the national grid to a standstill and low investment in the system. However, it is essential to preserve the national network as a power supply support to domestic and enterprise demand. To understand this scenario, we include an analysis of zero-rate demand growth. Under this hypothesis, a none smooth bifurcation appears related to a policy which involves the variation of the capacity charge. As a first significant result, we found that it is possible to preserve the investments in the market since, through the capacity charge parameter, the system dynamics can be controlled. Then, from a business approach, it is necessary to know the effects of the capacity charge as the strategic policy in the system generation price scheme.

**Keywords.**

Electricity market, modeling, simulation, complexity, non-smooth dynamic, supply and demand.

## 1. Introduction

To study the systems in the particular case of this document, we start with modeling of system dynamics. The concept of qualitative has had different connotations in the field of modeling. It would be expected that the information contained in the causal diagrams has this nature. That is not the meaning of qualitative that is discussed. The qualitative term refers to the different phenomena that a system exhibits, that is, the tendencies of its dynamic flow over time. For example, when in the system a level variable increases, decreases, oscillates or reaches a point of equilibrium. And, besides, if this dynamic is determined by a specific parameter (leverage points) or set of decision rules that affect it, boundary conditions, discontinuities, among others.

The dynamics of systems is structured so that it has a part of mathematical modeling and another that focuses on numerical methods of simulation. For this reason, a model based on systems dynamics is a mathematical object (Aracil and Gordillo, 1997). Hence, it deserves a mathematical analysis of its dynamics. From the above, it turns out that there is a wide range of strategies to model and simulate electricity markets (Ahmad et al., 2016, Foley et al., 2010, Gary and Larsen, 2000, Jebaraj and Iniyan, 2006, Ponzo et al., 2011; Ventosa et al., 2005). However, regarding the exploitation of these models, the contributions are lower, perhaps because of the interests with which the modeling and analysis were developed.

There is a wide range of strategies to model, simulate and formulate policy elements in electricity markets. If, on the contrary, the differential equations that emerge from them are explored, to a lesser extent reports will appear in this sense. Now, what is the need for this approach? According to the methodological synthesis proposed in (Valencia, 2016), the relevance of the qualitative analysis of models based on systems dynamics is recognized. For scientists who are familiar with the subject, they will know that formulating a system of differential equations that represent the complex behavior of an economic or social system is not as simple as for real physical systems. Specific variables that can be endogenous, or on the contrary, evaluate their effects exogenously to reach some conclusion, are indeed in the air. Well, if the formulation based on systems dynamics is used, arriving at this type of representation and obtaining a system of differential equations is more straightforward and more systematic.

In the case of this article, the interest focuses on the use of this dynamic wealth, to make decisions that involve elements of electric power policy. Therefore, in a systematic way it is possible to formulate models that start from a mental schema and use other tools that are not generally used for these cases of study. The implementation of asynchronous mappings in social systems is then proposed (Di Bernardo et al., 1997). The proposal that never before had been present to study the transients in dynamic social systems that model electricity markets. In this same sense, the implementation and analysis of stability of differential equations systems, despite its high degree of discontinuity and non-linearity, gives modelers a set of parameters that determine leverage points and shows which settings affect in a significant market dynamic.

Then research questions arise around the subject. What variables could induce a specific dynamic? And, if indeed, these leverage points represent, in a certain way, real information for the regulators. If the elements of energy policy are understood as control elements. Operators can then, based on systems dynamics models, that with a qualitative analysis provide additional information, formulate investment policies in the reliability charge instead of increasing the price of scarcity, for example. It is worthwhile to use in addition to the sensitivity analysis offered by the simulation packages; sensitivity analyzes that are structured as asynchronous mappings since for this type of systems it is much more interesting to explore what happens in the transient state.

The modeling of electricity markets is of great interest to the academic community, such as society in general. Especially since what happened in the 90s, the process of liberalization of the electricity market, as well as regularization, increases its analysis complexity. For example, (Gary and Larsen, 2000) argues that traditional econometric models do not provide market information and that in this conventional way it is not possible to represent their complex behavior. However, interest has also focused on deregulation, concerned about the effects that any external or political event could have on them. Following work such as (Cárdenas et al., 2015, Ford, 1999, Ochoa and van Ackere, 2009), there is an overview of the modelers' interest and the study of the energy market. These studies have analyzed the impact of different policies associated with investment decisions and the short and medium term effects that any change in the system could face.

## 2. Methodology

The complementarity between systems dynamics and mathematical analysis, with the help of the modern theory of nonlinear dynamic systems, allows establishing the qualitative behavior. However, sometimes, it is difficult to distinguish between qualitative and quantitative. Therefore, starting from a mental scheme, which is fundamentally the root of systems dynamics, it is possible to reach quantitative models that describe the system in question. In this way, it is possible to use non-linear modeling schemes of soft or discontinuous systems (PWS of its acronym in English Piece-Wise Smooth), which represent in a high degree the phenomena present in real situations. This is how the dynamics of systems allows, with a high level of detail, to study substantially complex aspects, which in the long run are no more than a set of soft and fragmented systems, carefully interconnected by functions or mathematical laws. The models based on systems dynamics are the translation of a type of mental model in the language of dynamic systems (Aracil, 1999).

To write the methodological synthesis proposed here, the methodological schemes offered by Aracil are considered in their document on Systems Dynamics (Aracil and Gordillo, 1997). In this one, it is shown in a general way how, from a mental scheme and using the theory of systemic thinking, it is possible to take advantage of a model based on systems dynamics. Also, another reference is the work of Sterman Business Dynamics (Sterman, 2000), where it is possible to identify different tools and systemic methods to validate and evaluate a model.

## 3. The problem definition

In this stage, a series of dynamic hypotheses are presented that reflect the influence of the different components of the system to be studied. For example, how the increase or decrease of a variable could affect other system variables, and what type of variables should be included in the model. In this phase, all part of a mental scheme. It is assumed then, five main variables that affect the behavior of a national electricity market. Already in work presented by Dyner et al. (Dyner, 2000), the structure of a national electricity market was defined, which includes the supply and demand side as the leverage points of the same. Likewise, the reserve margin of the system was set as the bridge that unites supply with demand and allows measuring the effects on the price that this signal would have. It is possible to understand this market, like any other market in which the availability estimated through the reserve margin affects the price and investment in the system, a scheme that remains valid today and you can see other ways to use it in (Cárdenas et al., 2015; Castañeda et al., 2015).

In this sense, it is necessary to evaluate what happens when the growth rate of demand k is equal to zero, the case study in this article. It was found that, under this condition, the variation of the reliability charge produces a discontinuous bifurcation, causing market equilibria to go from being real to virtual and, in the case that all are virtual, arise through a line of equilibria strange oscillatory behaviors.

## 4. Conclusions

From a mathematical point of view, it has been demonstrated that there is a non-soft bifurcation when the parameter b varies. In the qualitative analysis carried out throughout the work, it was possible to arrive at the detail of all the behaviors associated with each of the boundary conditions that determine the behavior of the system. On the other hand, the approximations and numerical schemes used were adequate to solve the problem. Besides, again, the theory of dynamic systems associated with soft systems by sections facilitated recognizing each of the phenomena present there. By the same token, when a point of equilibrium is real, efficiently using some method of numerical integration, one can understand its behavior.

Now, from systems dynamics and national electricity markets, the result is even more interesting. First, there is a discontinuous bifurcation for a national electricity market, so far there are no reports in the literature on this. And second, under the conditions in which the bifurcation appears, for the operators it could mean the permanence response of a specific technology if the growth rate of demand is zero. That is, the conditions under which the investment in the market tends to the point of particular equilibrium are known.

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